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BEAM-BASED CALIBRATION OF SEXTUPOLE MAGNET DISPLACEMENT WITH BETATRON TUNE SHIFT

S. Takano[†1], T. Fujita, M, Takao, K. Fukami[1], M, Masaki, T, Watanabe[1], JASRI, Hyogo, Japan
H. Maesaka[2], K. Soutome[2], T. Hiraiwa, H. Tanaka, RIKEN SPring-8 Center, Hyogo, Japan
K. Ueshima, QST, Miyagi, Japan
[1]also at RIKEN SPring-8 Center, Hyogo, Japan
[2]also at JASRI, Hyogo, Japan

Abstract

The alignment of sextupole magnets is one of the critical issues for the upcoming 4th generation light sources and future colliders to ensure enough dynamic aperture for stable operation and minimize deterioration of beam quality. We propose a beam-based calibration (BBC) method for the sextupole magnet displacement by observing the betatron tune shift. The beam position that makes the horizontal and vertical betatron tunes invariant to the sextupole strength marks the magnet center. The key is to increase the betatron coupling so that the vertical displacement of a sextupole from the beam results in a tune shift large enough for the calibration. The feasibility studies at SPring-8 successfully demonstrated the principle for calibrating both horizontal and vertical sextupole displacements in quantitative agreement with the theory.

INTRODUCTION

The alignment of sextupole magnets is one of the critical issues for the upcoming 4th generation light sources and future colliders. The misalignment and the beam offset in sextupoles should be within a few 10 µm to ensure enough dynamic aperture for stable operation and minimize deterioration of beam quality. A sextupole magnet horizontally displaced from the beam exerts a normal quadrupole (Q) field, and a vertically displaced one a skew Q field. We propose a beam-based calibration (BBC) method to measure the displacement of a sextupole magnet by observing the betatron tune shift. The beam position that makes the horizontal and vertical betatron tunes invariant to the sextupole strength marks the magnet center. We studied experimentally the feasibility of the proposed BBC for the sextupole magnet on the SPring-8 storage ring and successfully demonstrated the principle for calibrating both the horizontal and vertical displacements. In this paper, we overview the theoretical background related to the betatron tune-based BBC and elaborate on the results of feasibility studies at SPring-8.

THEORETICAL BACKGROUND

Magnetic Field of a Sextupole

Suppose that a beam passes through a sextupole magnet with offset (x_0, y_0) from the center of the magnetic field. When each particle in the beam moves (Δx, Δy) around this point, the magnetic field at the destination point is expressed in the forms

$$B_x = B''(x_0 + \Delta x)(y_0 + \Delta y), \tag{1}$$

$$B_y = \frac{B''}{2}\{(x_0 + \Delta x)^2 - (y_0 + \Delta y)^2\}, \tag{2}$$

where B" is the gradient of the sextupole magnet. Eqs. (1) and (2) can be rewritten in the forms

$$B_x = B''x_0y_0 + B''x_0\Delta y + B''y_0\Delta x + B''\Delta x\Delta y, \tag{3}$$

$$B_y = \frac{B''}{2}(x_0^2 - y_0^2) + B''x_0\Delta x - B''y_0\Delta y + \frac{B''}{2}((\Delta x)^2 - (\Delta y)^2). \tag{4}$$

The second and third terms in Eqs. (3) and (4) represent the normal and skew Q components in the magnetic field, respectively. Therefore, in a sextupole magnet, the beam passing through with a horizontal offset, $x_0 \neq 0$, undergoes a normal Q magnetic field, and the one passing through with a vertical offset, $y_0 \neq 0$, undergoes a skew Q field. The skew Q field in the sextupole magnet drives the linear coupling of the betatron motion that adds to the indigenous coupling driving term of the whole storage ring.

Betatron Tune Shift by a Displaced Sextupole

For the coupled betatron motion, if we define the difference between the fractional parts of unperturbed betatron tunes ν_x and ν_y as

$$\Delta \equiv \nu_x - \nu_y - q \tag{5}$$

with q an integer, the eigentunes in the normal coordinate ν_u and ν_v, which are observed in actual measurements instead of ν_x and ν_y, can be expressed as

$$\nu_{u,v} = \nu_{x,y} \mp \frac{1}{2}\left(\Delta - \sqrt{\Delta^2 + |C|^2}\right), \tag{6}$$

and

$$\nu_{u,v} = \nu_{x,y} \mp \frac{1}{2}\left(\Delta + \sqrt{\Delta^2 + |C|^2}\right), \tag{7}$$

for $\Delta \geq 0$ and $\Delta < 0$, respectively, where C is the coupling driving term (complex number) for the whole ring [1]. In what follows, we will address the case $\Delta < 0$ as in the studies reported in this paper.

† takano@spring8.or.jp

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Suppose the sextupole gradient is varied by ΔB" as the beam passes at (x_0, y_0) in the sextupole. Then, the change in the horizontal and vertical betatron tunes can be expressed as

$$\Delta\nu_x = +\frac{\beta_x(s)}{4\pi}\frac{\Delta B''L}{[B\rho]}x_0 - \frac{1}{2}\sqrt{\Delta^2+|C_0+\delta C|^2} + \frac{1}{2}\sqrt{\Delta^2+|C_0|^2}, \quad (8)$$

$$\Delta\nu_y = -\frac{\beta_y(s)}{4\pi}\frac{\Delta B''L}{[B\rho]}x_0 + \frac{1}{2}\sqrt{\Delta^2+|C_0+\delta C|^2} - \frac{1}{2}\sqrt{\Delta^2+|C_0|^2}, \quad (9)$$

where C_0 is the initial coupling driving term for the whole ring. The first term in Eqs. (8) and (9) represents the tune shift due to a change in the focusing of the normal Q field of a horizontally displaced sextupole. The second term in Eqs. (8) and (9) is due to a change in the coupling δC brought about by the skew Q field of a vertically displaced sextupole. The additional driving term δC is given by

$$\delta C = \frac{1}{2\pi}\frac{\Delta B''L}{[B\rho]}y_0 \sqrt{\beta_x(s)\beta_y(s)}e^{i[\phi_x(s)-\phi_y(s)-2\pi\Delta\cdot s/L_0]} \quad (10)$$

with the ring circumference L_0, and the betatron functions and phase advances at the sextupole β_x, β_y, ϕ_x, and ϕ_y. Eqs. (8), (9), and (10) illustrate that the magnetic center of the sextupole magnet can be marked as the beam position where both the horizontal and vertical tunes are invariant to the sextupole field strength. To lowest order of $|\delta C|$, Eqs. (9) and (10) can be rewritten as

$$\Delta\nu_x \fallingdotseq +\frac{\beta_x(s)}{4\pi}\frac{\Delta B''L}{[B\rho]}x_0 - \frac{\sqrt{\beta_x(s)\beta_y(s)}}{4\pi}\frac{\Delta B''L}{[B\rho]}\frac{|C_0|cos(\phi_0-\phi_1)}{\sqrt{\Delta^2+|C_0|^2}}y_0, \quad (11)$$

$$\Delta\nu_y \fallingdotseq -\frac{\beta_y(s)}{4\pi}\frac{\Delta B''L}{[B\rho]}x_0 + \frac{\sqrt{\beta_x(s)\beta_y(s)}}{4\pi}\frac{\Delta B''L}{[B\rho]}\frac{|C_0|cos(\phi_0-\phi_1)}{\sqrt{\Delta^2+|C_0|^2}}y_0, \quad (12)$$

where ϕ_0 and ϕ_1 are arguments of C_0 and δC, respectively.

Betatron Tune Based Sextupole Displacement Calibration

Putting $\Delta\nu_x = \Delta\nu_y = 0$ in Eqs. (8) and (9) or in (11) and (12), two loci of fixing points for ν_x and ν_y can be obtained. If we suppose the horizontal and vertical betatron functions at the target sextupole magnet are different, the two loci intersect at only one point at the sextupole center.

In the small coupling limit, $|C_0| \to 0$, information on the vertical center of the sextupole is missing in the tune shift. In that case, we can find the horizontal center of the sextupole by moving the beam horizontally regardless of the vertical beam offset and by observing either the horizontal or vertical tune. For real rings with a finite coupling, the key is to increase the betatron coupling so that the vertical displacement of a sextupole from the beam results in a tune shift large enough for calibration. In our feasibility studies at SPring-8, we increased the coupling as described below.

FEASIBILITY STUDIES AT SPring-8

We conducted feasibility studies on the betatron tune-based BBC of sextupole magnet displacement on the SPring-8 storage ring, an 8 GeV photon source electron ring with a 1436 m circumference. For the target of the BBC, a sextupole magnet placed in one of the 30-m-long straights of the ring [2] with a beam position monitor (BPM) head nearby was employed. The beam position at the target sextupole was controlled by using local orbit bumps. There were two other sextupoles involved in the orbit bumps that are used in the normal operation of SPring-8. They were switched off temporarily during the experiments.

Beam Monitoring

Equally spaced 21 electron bunches were stored on the storage ring for the BBC study. The BPM nearby the target sextupole was read by a recently developed MicroTCA.4-based electronics [3]. Fast COD data were sampled at 10 kHz with sub-micron resolution [3]. Two out of the stored 21 bunches were subject to real-time tune monitoring, one for the horizontal tune and the other for the vertical one. Two sets of bunch-by-bunch feedback and diagnostics signal processor iGP12 of Dimtel, Inc. [4] were employed. Single bunch phase tracker capability integrated into the processor provided real-time tune monitoring with ~ 10^{-5} precisions at a 5 Hz sampling during the experiments.

Coupling Control with Skew Q Magnets

To obtain effective data sensitive to the vertical displacement of the sextupole magnet relative to the beam, we temporarily increased the betatron coupling by using skew Q magnets integrated into the storage ring. The overall coupling driving term C_T can be expressed as

$$C_T = C_0 + \Delta C_{SkQ} \quad (13)$$

where C_0 is the original coupling driving term for the whole ring and ΔC_{SkQ} is the driving term added by the skew Q magnets. First, we searched the argument ϕ_0 of C_0 by observing the projected beam size σ_y varying the argument ϕ_{SkQ} of ΔC_{SkQ} keeping the magnitude of ΔC_{SkQ} at a constant value. The beam size σ_y can be related to the argument ϕ_{SkQ} by a formula

$$\frac{\sigma_y^2}{\beta_y} = \frac{\frac{1}{2}|C_T|^2}{\Delta^2+|C_T|^2}\varepsilon_0 \approx \frac{\frac{1}{2}\left(|C_0|^2+2|C_0||\Delta C_{SkQ}|cos(\phi_{SkQ}-\phi_0)+|\Delta C_{SkQ}|^2\right)}{\Delta^2}\varepsilon_0, \quad (14)$$

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where ε_0 is the beam emittance. The value of ϕ_{SkQ} maximizing the magnitude $|C_T|$ was found to be 27˚ and the argument ϕ_0 was experimentally determined as 27˚ as shown in Fig.1. Hereafter, we fixed the argument ϕ_{SkQ} at 27˚. Second, we observed the projected beam size σ_y varying the magnitude of ΔC_{SkQ} to evaluate the magnitude of C_0. The value of $|\Delta C_{SkQ}|$ minimizing the vertical σ_y was found to be -0.008 and the magnitude $|C_0|$ was experimentally determined to be 0.008 as shown in Fig.2. Finally we set the $|\Delta C_{SkQ}|$ to the maximum available value of 0.05, which lead to a magnitude of the total coupling driving term $|C_T|$ of 0.058.

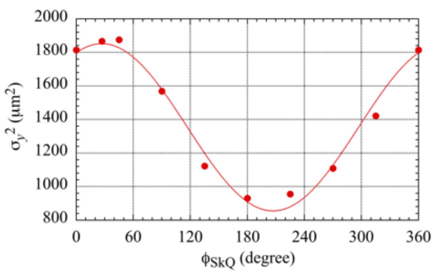

Figure 1: Search for ϕ_{SkQ} maximizing σ_y.

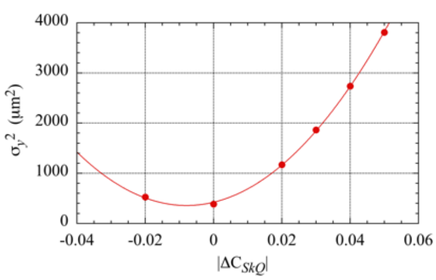

Figure 2: Search for $|\Delta C_{SkQ}|$ minimizing σ_y.

The sensitivity to the vertical displacement of the sextupole magnet relative to the beam is proportional to a factor, $|C_0|/\sqrt{\Delta^2 + |C_0|^2}$, as shown in Eqs. (11) and (12). Besides the coupling control described above, the magnitude of Δ, the difference between the fractional parts of unperturbed betatron tunes ν_x and ν_y, was decreased to obtain extra gain for the sensitivity to the vertical displacement. The original betatron tune (ν_x, ν_y) was (41.14, 19.325). The vertical tune was changed to 19.225 temporarily for the experiments. The enhancement in the factor, $|C_0|/\sqrt{\Delta^2 + |C_0|^2}$, is calculated to be 13. Sensitivity to the vertical offset was improved by a factor of more than 10.

Proof-of-Principle Experiments

The measurement procedures in our proof-of-principle experiments for the sextupole BBC are as follows,

- Step 1: Set a vertical orbit bump.
- Step 2: Sweep the horizontal beam position with a local orbit bump monitoring the beam position and the betatron tune under two conditions: target sextupole magnet OFF and ON.
- Step 3: Set other vertical orbit bumps, and repeat the measurements in Step 2 for each set of the vertical orbit bump.
- Step 4: For each set of the vertical orbit bump, find beam positions (x_1, y_1) and (x_2, y_2) where the horizontal and vertical betatron tunes are invariant, respectively, for the sextupole OFF and ON. The intersection of the loci of (x_1, y_1) and (x_2, y_2) marks the center of the target sextupole magnet

Results

Examples of data obtained in Step 2 of the proof-of-principle experiments for the sextupole BBC are shown in Fig. 3.

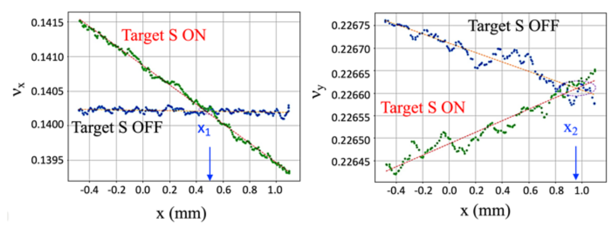

Figure 3: Examples of data obtained in Step 2 of the proof-of-principle experiments for the sextupole BBC corresponding to a vertical orbit bump of +0.1 mm.

The sextupole center is the beam position where both the horizontal and vertical betatron tunes are invariant to the magnet strength as shown in Eqs. (8) and (9). Measured horizontal tune fixing points (x_1, y_1) and vertical tune fixing points (x_2, y_2) are shown in Fig. 4. The

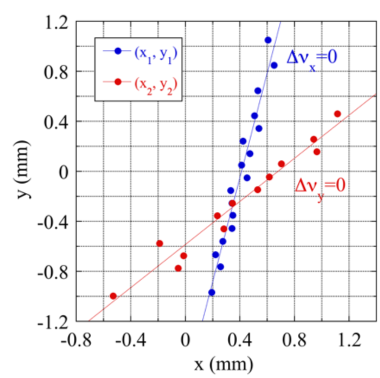

Figure 4: Measured horizontal tune fixing points (x_1, y_1) and vertical tune fixing points (x_2, y_2). The intersection of fitted loci of (x_1, y_1) and (x_2, y_2), respectively, marked the center of the target sextupole magnet.

intersection of fitted loci for (x_1, y_1) and (x_2, y_2), respectively, successfully marked the center of the target sextupole magnet. The beam position (x, y) corresponding to the center of the target sextupole magnet was determined to be $(0.347 \pm 0.014, -0.288 \pm 0.034)$ in millimeters where the precisions were evaluated from the residuals of the fit. The specified beam position also figures the BPM offset to the target sextupole.

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To compare the results in Fig. 4 quantitatively with the theory given by Eqs. (8) and (9), the betatron function at the target sextupole was calibrated by using the observed tune shift due to the normal Q field in the target sextupole corresponding to the first term in Eqs. (8) and (9). The horizontal betatron function β_x was $32.16 \pm 0.17\ m$, and the vertical betatron function β_y was $6.41 \pm 0.12\ m$. The measured horizontal tune fixing points (x_1, y_1) and vertical tune fixing points (x_2, y_2) are shown in Fig. 5 after subtracting the BPM offset together with the loci of tune fixing points calculated with the experimentally calibrated betatron function at the target sextupole. The measured fixing points for ν_x and ν_y were in good agreement with that calculated theoretically as shown in Fig. 5.

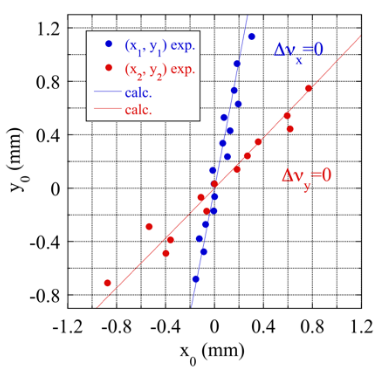

Figure 5: Measured horizontal and vertical tune fixing points (x_1, y_1) and (x_2, y_2), respectively, after subtracting the BPM offset compared with the calculations by Eqs. (8) and (9).

The principle of the betatron tune-based BBC for the sextupole displacement was successfully demonstrated in quantitative agreement with the theory. The precision of the measured sextupole center was a few tens of microns.

ISSUES AND CHALLENGES

We found that the vertical tune unexpectedly changed depending on the horizontal beam position even when the target sextupole was off, as shown in Fig. 3. Influences of the residual field in the sextupoles involved in the orbit bumps and the orbit bump leakages need to be investigated.

While the experimentally evaluated horizontal betatron function β_x of 32.2 m was consistent with the design value of 34.5 m, the experimental β_y of 6.4 m was smaller than the design of 10.5 m. Detailed examinations of the distortions of betatron functions over the storage ring would be necessary.

To apply the betatron tune-based BBC to the entire sextupole magnets in a storage ring for practical use, it is necessary to complete the measurement in a short time. Future studies will be considered employing faster beam orbit manipulation, i.e. by AC exciting the steering magnets and faster betatron tune tracking.

SUMMARY

A beam-based calibration (BBC) method with the betatron tune shift for the sextupole magnet displacement is proposed. The feasibility studies at SPring-8 successfully demonstrated the principle for both horizontal and vertical sextupole displacements in quantitative agreement with the theory. The measurement resolution of 10 µm will be feasible with further improvements. Issues on the observed horizontal position-dependent vertical tune variation for the target sextupole switched off and the vertical betatron function at the target sextupole need to be resolved by future studies. To apply to the entire sextupole magnets in a storage ring for practical use, it is necessary to complete the measurement in a short time. Future studies will be considered employing faster beam orbit manipulation and betatron tune tracking.

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